

ML²S: Minimum Length Link Scheduling Under Physical Interference Model

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Abstract—We study a fundamental problem called Minimum Length Link Scheduling (ML²S) which is crucial to the efficient operations of wireless networks. Given a set of communication links of *arbitrary* length spread and assume each link has one unit of traffic demand in wireless networks, the problem ML²S seeks a schedule for all links (to satisfy all demands) of minimum number of time-slots such that the links assigned to the same time-slot do not conflict with each other under the physical interference model. In this paper, we will explore this problem under three important transmission power control settings: linear power control, uniform power control and arbitrary power control. We design a suite of new and novel scheduling algorithms and conduct explicit complexity analysis to demonstrate their efficiency. Our algorithms can account for the presence of background noises in wireless networks. We also investigate the fractional case of the problem ML²S where each link has a fractional demand. We propose an efficient greedy algorithm of the approximation ratio at most $(K+1)^2\omega$.

I. INTRODUCTION

Wireless link scheduling plays a critical role in wireless networks and has been extensively studied in the literature [10], [11], [20]. The task of link scheduling (or medium access control) is challenging due to the simultaneous presence of two characteristics: wireless interferences among concurrent transmissions, and the need for practical distributed implementation with small communication overhead and time complexity. The task will be more challenging if there is a request for the performance guarantee of a link scheduling algorithm. It is well known that a number of scheduling problems (*e.g.*, maximum throughput scheduling) become NP-hard when considering wireless interferences, while their counter-parts are solvable in polynomial time for wired networks. Thus, the scheduling algorithms for wireless networks often rely on heuristics that approximately optimize the throughput.

As we know, the signal interferences cast significant effect on the data throughput (or capacity region) that any scheduling algorithms can achieve. Thus, we need to model and take care of wireless interferences carefully to ensure the correctness of algorithm design. Wireless link scheduling under various graph-based interference models (*e.g.*, the protocol interference model, CTS / RTS model,

K -hop interference model) have been extensively studied in the literature [14], [21]. However, there are not many positive results on a series of problems related to link scheduling under the physical interference model [16], [17]. These problems receive great research interests recently in network community for its accurate capture of signal interferences. Under the physical interference model, we will model a successful transmission as follows: a signal is received successfully if the Signal to Interference-plus-Noise Ratio (SINR) is above a certain threshold. This threshold depends on hardware and coding method.

By using this practical interference model, a successful transmission also accounts for interferences generated by transmitters located far away, this fact greatly differs that in graph-based interference models. Therefore, traditional algorithms for protocol interference model, RTS/CTS model *et al.* cannot be directly applied here. We will focus on the physical interference model and address a fundamental problem for wireless link scheduling called Minimum Length Link Scheduling (ML²S) or Shortest Link Scheduling. Given is a set of links of arbitrary length diversity (or spread), assume each link has one unit demand, the objective is to schedule all links (or to satisfy all demands) within a minimum number of time-slots; at the same time, the links scheduled in the same time-slot do not conflict with each other under the physical interference model. This problem has several variations, one important variation is to consider the problem under different transmission power control settings. We will focus on three general power assignment settings: linear power control, uniform power control, and arbitrary power control.

Linear Transmission Power Control [24]: Each link l (or its corresponding sender) is assigned with a transmission power $c \cdot \|l\|^\beta$, where c is a constant and $0 < \beta \leq \kappa$ is a constant. As a long communication link requires more transmission power, this power control setting is usually energy-efficient. Note that linear power assignment is a representative of the *oblivious* power assignment family where the transmission power of each communication link only depends on its length.

Uniform Transmission Power Control [10], [11]: Each link is assigned with an identical transmission power. The uniform power setting has been widely adopted in the literature while it is notoriously hard to achieve a constant approximation for several variants of the link scheduling problems under this setting; for instance, to find constant approximations for both problems ML^2S and *Maximum Weight Independent Set of Links (MWIL)* remain open. Given a set of communication links, the problem MWIL seeks a subset of maximum weight, such that all links in this subset can transmit concurrently.

Arbitrary Transmission Power Control: Each link is assigned with a transmission power of arbitrary value. Clearly, this assignment is the most general.

Our main contribution is as follows. We study the problem ML^2S under the physical interference model with different transmission power assignments. Under the linear power control setting, we are the first to propose algorithm design for the problem ML^2S and prove the NP-hardness of this problem. We use a partition strategy to find multiple sets of well-separated links and combine them to form a schedule. We prove that the solution returned by our algorithm satisfies the wireless interference constraint and can achieve constant approximation bound compared to the optimum one. Under the uniform power control setting, we propose two algorithms that can achieve an approximation bound of $\min\{O(\log \max\{f(l_i)\}), O(\log n)\}$ for the problem ML^2S . We also present an efficient linear programming-based method for this problem under the arbitrary power control setting. On the other hand, we investigate the fractional case of the problem ML^2S where each link has a fractional demand. We propose an efficient greedy algorithm and show that its approximation ratio is at most $(K+1)^2\omega$.

The rest of the paper is organized as follows. Section II formulates the problem ML^2S and presents the complexity analysis. Section III, IV and V are devoted to the algorithm design for the problem ML^2S under three transmission power control settings respectively. Section VI deals with the fractional case. Section VII outlines the related work. Finally, Section VIII concludes the paper.

II. NETWORK MODEL

Assume we are given a set of links A where the networking nodes V (i.e., sending and receiving nodes of all links) lie in plane, each node has a transmission power upper-bounded by P_{\max} (or simply P). Denote the Euclidean distance between a pair of nodes u and v by $\|uv\|$.

Let r be the smallest distance between any pair of nodes in V . The path-loss over a distance l is $\eta l^{-\kappa}$, where κ is *path-loss exponent* (a constant greater than 2 and 5 depending on the wireless environment), and η is the *reference loss factor*. Since the path-loss factor over the distance r is less than one, we have $\eta < r^\kappa$.

We will consider three power assignments. In the uniform power assignment, each link is assigned with the same transmission power [10], [11]. In arbitrary power assignment, the power of each link can be arbitrary. In an *oblivious power control setting* [8], a node u transmits to another node v always at a power depending on the length of the link. In this work, we will assume the power for link \vec{uv} is $c\|uv\|^\beta$ for some constants $c > 0$ and $0 < \beta \leq \kappa$. When $\beta = \kappa$, this implies a *linear power assignment*. This assumption implicitly imposes an upper bound on the distance between a pair of nodes which directly communicate with each other: For u to be able to directly communicate with v , we must have $c\|uv\|^\beta \leq P$ and hence $\|uv\| \leq (P/c)^{1/\beta}$.

Let ξ be the noise power, and σ be the *signal to interference and noise ratio (SINR)* threshold for successful reception. Then, in the absence of interference, the transmission by a node u can be successfully received by another node v if and only if $\frac{c\|uv\|^\beta \cdot \eta \|uv\|^{-\kappa}}{\xi} \geq \sigma$ which is equivalent to $\|uv\|^{\kappa-\beta} \leq \frac{c\eta}{\sigma\xi}$. Note that when $\|uv\|^{\kappa-\beta} = \frac{c\eta}{\sigma\xi}$, link uv can only transmit alone since any other link will conflict with uv . Thus we can disregard these links in A and assume that

$$\|uv\|^{\kappa-\beta} < \frac{c\eta}{\sigma\xi}$$

Therefore, the set A of communication links consists of all pairs (u, v) of distinct nodes satisfying that $\|uv\|^\beta \leq P/c$ and $\|uv\|^{\kappa-\beta} < \frac{c\eta}{\sigma\xi}$. Let R be the maximum length of the links in A . Then,

$$R^\kappa = R^\beta \cdot R^{\kappa-\beta} < \frac{P}{c} \cdot \frac{c\eta}{\sigma\xi} = \frac{P\eta}{\sigma\xi} < \frac{P}{\sigma\xi} r^\kappa.$$

Hence,

$$\frac{R}{r} < \left(\frac{P}{\sigma\xi}\right)^{1/\kappa}.$$

For a set of links \mathcal{L} , the *length diversity* (or link spread) is defined as the $\log \frac{\max\{\|l_i\| : l_i \in \mathcal{L}\}}{\min\{\|l_i\| : l_i \in \mathcal{L}\}}$. Given an input links of arbitrary length spread, a set I of links in A is said to be *independent* if and only if all links in I can transmit successfully at the same time under the physical interference model with oblivious power assignment, i.e., the SINR of each link in I is above σ . We denote by \mathcal{I} the collection of independent sets of links in A . Given a set of input links A , assume each link has one unit demand, the problem **Minimum Length Link Scheduling (ML^2S)** or Shortest Link Scheduling seeks a schedule (partitioning the input links into multiple disjoint subsets) of minimum length such that each subset of links are independent.

A fractional link schedule is a set

$$S = \{(I_j, \gamma_j) : 1 \leq j \leq k\}$$

with $I_j \in \mathcal{I}$, and $\gamma_j \in \mathbb{R}^+$ for each $1 \leq j \leq k$. The value $\sum_{j=1}^k \gamma_j$ is referred to as the length of the schedule S . We

Input : Set of links $A = \{l_1, l_2, \dots, l_n\}$.
Output: Multiple independent sets
 $S = \{S_1, S_2, \dots, S_l\}$ of links
Set K according to Equation (1);
 $k \leftarrow 0$;
for $r = 0, \dots, K$ and $s = 0, \dots, K$ **do**
 while **TRUE** **do**
 $k = k + 1$;
 for $i, j \in \mathbb{Z}$ and the cell $g_{i,j}$ contains links
 from A **do**
 if $i \bmod (K+1) = r$ and
 $j \bmod (K+1) = s$ **then**
 select one link whose sender lies
 within $g_{i,j}$;
 all the selected links form a set S_k ;
 if $S_k = \emptyset$ **then**
 exit the while loop;
 $A \leftarrow A \setminus S_k$;
return S_1, S_2, \dots, S_l , here $l = k - 1$.

Algorithm 1: Scheduling for ML²S

define the link load function $c_S \in \mathbb{R}_+^A$ supported by the fractional schedule S as

$$c_S(e) = \sum_{j=1}^k \gamma_j \cdot |I_j \cap \{e\}|, \forall e \in A$$

Suppose we are give a set of links, each link e is associated with a fractional demand $d(e)$, the problem **Minimum Length Fractional Link Scheduling (FML²S)** or Shortest Fractional Link Scheduling seeks a fractional link schedule (each subset of links is an *independent set of links*), of shortest length, such that for each link e , the demand $d(e) = c_S(e)$. Please refer to [21] for a complete and detailed definition on the fractional link schedule.

Complexity of the problem ML²S: We will prove that the problem ML²S is NP-hard. The Partition problem has been proved to be NP-hard [15], thus we only need to reduce the Partition problem to ML²S.

Theorem 1: The Partition problem is reducible to the problem ML²S in polynomial time (The proof is available in the appendix).

III. LINEAR POWER CONTROL

In this section, we propose a distributed constant-approximation algorithm for the ML²S problem under the linear power control model.

Based on the definition of physical interference model, if we select a subset of geographically well-separated links and let only these links transmit simultaneously, we can ensure that the interference on each link is bounded. Using this property, we can schedule links in A based on a partition scheme of the plane (Fig. 1)

For any link $l_i \in A$, we know $\|l_i\| \leq R$. Let $\ell = R/\sqrt{2}$. The vertical lines $x = i \cdot \ell$ for $i \in \mathbb{Z}$ and horizontal lines $y = j \cdot \ell$ for $j \in \mathbb{Z}$ partition the planes into half-open, half-closed grids of side length ℓ (here \mathbb{Z} represents the integer set):

$$\{[i\ell, (i+1)\ell) \times [j\ell, (j+1)\ell) : i, j \in \mathbb{Z}\}.$$

Based on a partition of the plane, we define a large-block as a square which consists of $K \times K$ cells (grids). Here we

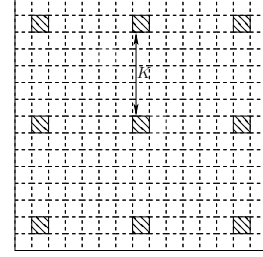


Fig. 1: A partition of the plane into cells, each large-block consists of $K \times K$ cells.

let

$$K = \lceil \sqrt{2} \left((4\tau)^{-1} \left(\sigma^{-1} - \xi(c\eta)^{-1} R^{\kappa-\beta} \right) \right)^{-1/\kappa} + \sqrt{2} \rceil \quad (1)$$

Generally, K 's value depends on R , we can see that when $\beta = \kappa$ (which is exactly the linear power assignment), we have,

$$K = \lceil \sqrt{2} \left((4\tau)^{-1} \left(\sigma^{-1} - \xi(c\eta)^{-1} \right) \right)^{-1/\kappa} + \sqrt{2} \rceil$$

which is a constant independent of R .

For each time-slot, we let an independent set of links transmit, which is formed by picking at most one link from only the cell lying in the same relative location from every large-block. We proceed until all links have been picked already. The details of our partition-based scheduling algorithm is shown in Algorithm 1. The correctness of the algorithm follows from Lemma 1. Note that:

- 1) the output by Algorithm 1 (the total number of time-slots needed) depends on the maximum number of links located in a large-blocks.
- 2) we do not keep any monotone ordering on the cardinality of link set scheduled in each time-slot. In other words, for i -th time-slot, we may schedule more links than the $(i-1)$ -th time-slot ($i \in \mathbb{N}$).

We first verify the correctness of our algorithm: each subset of links outputted by the proposed algorithm are independent (Lemma 1).

For all $i, j \in \mathbb{Z}$, we denote A_{ij} to be the set of links in A whose senders lie in the grid

$$[i\ell, (i+1)\ell) \times [j\ell, (j+1)\ell).$$

Lemma 1: Consider any two nonnegative integers k_1 and k_2 which are at most K . Suppose I is a set of links satisfying that for each $i, j \in \mathbb{Z}$, $|I \cap A_{ij}| \leq 1$

if $i \bmod (K+1) = k_1$ and $j \bmod (K+1) = k_2$ and $|I \cap A_{ij}| = 0$ otherwise. Then, I is independent (The proof is available in the appendix).

We then calculate the approximation ratio of the proposed algorithm. The main idea is as follows: we first estimate the number of time-slots needed for Algorithm 1 and derive an upper-bound. Then, we will compute a lower-bound on the number of time-slots required for any algorithm to schedule links lying inside a single large-block, thus a lower-bound for the length of any schedule to transmit all input links. By comparing these upper-bound and lower-bound, we can compute the approximation ratio of the proposed algorithm.

Based on the partition scheme (Fig. 1), we assume the maximum number of senders from A lying inside any small cell is B . Then, we have the following lemma on the upper-bound of the number of time-slots needed for Algorithm 1.

Lemma 2: The proposed algorithm costs at most $(K+1)^2 \cdot B$ time-slots.

Proof: As shown in line 3 of Algorithm 1, there are at most $(K+1)^2$ while loops. After each iteration of a while loop, the links whose sender lies inside each cell will decrease by at least one. Thus, the number of B iterations for the while loop is at most B . In total, Algorithm 1 costs at most $(K+1)^2 \cdot B$ time-slots. ■

Next, we calculate the lower-bound of any algorithm for ML²S that is the minimum possible length for any algorithm.

Let

$$\omega = \left\lceil \frac{2^\kappa P}{\sigma^2 \xi} + 1 \right\rceil.$$

Lemma 3: For any $I \in \mathcal{I}$ and any $i, j \in \mathbb{Z}$, $|I \cap A_{ij}| \leq \omega$.

Proof: Let $I_{ij} = I \cap A_{ij}$. Assume $a = (u, v)$ be the shortest link in I_{ij} , consider any link $a' = (u', v')$ in I_{ij} other than a , the distance between the sender u' and v satisfies

$$\|u'v\| \leq \|u'u\| + \|uv\| \leq \|\sqrt{2}\ell\| + \|uv\| \leq 2R,$$

The SINR at a from all other links in I_{ij} is at most

$$\begin{aligned} & \frac{c \|a\|^\beta \cdot \eta \|a\|^{-\kappa}}{\sum_{a' \in I_{ij} \setminus \{a\}} c \|a'\|^\beta \cdot \eta \|u'v\|^{-\kappa}} \\ & \leq \frac{\|a\|^{-\kappa}}{\sum_{a' \in I_{ij} \setminus \{a\}} \|u'v\|^{-\kappa}} \\ & \leq \frac{r^{-\kappa}}{(|I_{ij}| - 1)(2R)^{-\kappa}} \end{aligned}$$

Since $\frac{r^{-\kappa}}{(|I_{ij}| - 1)(2R)^{-\kappa}} \geq \sigma$, we have $|I_{ij}| \leq \frac{2^\kappa R^\kappa}{\sigma} + 1 < \frac{2^\kappa P}{\sigma^2 \xi} + 1 = \frac{2^\kappa P}{\sigma^2 \xi} + 1$. ■

Since there exists one cell that contains B senders, by Lemma 3, this fact immediately implies a lower-bound on the number of time-slots required for any algorithm to schedule links lying inside a large-block, and thus a lower-bound for any algorithm to schedule all input links.

Corollary 1: Any valid schedule for ML²S has a length at least $\frac{B}{\omega}$.

Theorem 2: The approximation ratio of our algorithm for ML²S is at most $(K+1)^2 \omega$.

Proof: Since our algorithm has a length at most $(K+1)^2 \cdot B$, at the same time, any valid schedule for ML²S has a length at least $\frac{B}{\omega}$, thus the approximation ratio of the proposed algorithm can be bounded by $(K+1)^2 \cdot \omega$. So, the theorem holds. ■

IV. UNIFORM POWER CONTROL

In this section, we design a scheduling for the problem ML²S under the uniform transmission power setting.

A. First Algorithm

For each link $l_i \in A$, we first introduce a concept of *conflict range factor* which is defined as

$$f(l_i) = \frac{1}{1 - \|l_i\|/R}$$

The conflict range factor can indicates how far should a set of links of similar lengths be separated to ensure concurrent transmissions. Observe that the concept of *relative interference* as defined in [20] is closely related the conflict range factor.

We then calculate the conflict range factor of each link and partition the links into $\log \max\{f(l_i) : l_i \in A\}$ groups. The first group consists of the links with the conflict range factor at most one. The i -th ($2 \leq i \leq \log \max\{f(l_i) : l_i \in A\}$) group G_i consists of the links with the conflict range factor lying in $[2^{i-1}, 2^i]$. For each group G_i , we can use partition and shifting to find a schedule for all member links, which consists multiple scheduling sets. We can output the union of all scheduling sets for all groups, as our solution. According to Lemma 1 in [20], we can verify the correctness of our algorithm easily.

We next derive an approximation bound on the performance of our algorithm.

Theorem 3: The number of scheduling set resulted from the proposed algorithm is at most $\log \max\{f(l_i) : l_i \in A\}$ times the optimum solution.

Proof: Considering the group G_i which costs the maximum number of time-slots in our algorithm. Assume the number of time-slots to transmit all links in G_i by our algorithm is k . Thus our algorithm costs at most $k \cdot \log \max\{f(l_i)\}$ time-slots. Since all links in G_i has similar conflict range factor, we can verify that any feasible schedule would take at least $O(k)$ time-slots to transmit all links in this group G_i . Thus the approximation bound of our algorithm is $O(\log \max\{f(l_i)\})$. ■

B. Second Algorithm

Our second algorithm consists in iteratively computing a maximum independent set of links by using the algorithm in Table 1 in [20]. We then transmit each independent set of links in a separate time-slot, and the remaining links are repeatedly used as input to the algorithm in Table 1 in [20]. The procedure continues until all links in A have been scheduled. The correctness of the obtained schedule has been proved as the algorithm in Table 1 in [20] indeed outputs an independent set of links. It is trivial to show that the approximation bound of the second algorithm is $O(\log n)$.

We make a note that Goussevskaya *et al.* [11] made the first effort on developing a $O(\log n)$ for the problem ML^2S under the uniform transmission power control setting. However, as observed in [23], the claimed constant approximation bound and its proof (Lemma 4.5 in [11]) are valid only when the background noise is zero; thus their result becomes baseless. Despite of this effort in [11], the existence of a $O(\log n)$ -approximation algorithm for ML^2S under the uniform transmission power control setting remains open.

To sum up, we can achieve an approximation bound of $\min\{O(\log \max\{f(l_i)\}), O(\log n)\}$ for the problem ML^2S under the uniform transmission power control setting.

V. ARBITRARY POWER CONTROL

This section presents the link scheduling under the physical interference model with arbitrary power setting. We will borrow the idea of cover inequalities in [3] of solving the problem maximum independent set of links to our setting. The technique of cover inequalities has found applications for solving the 0-1 knapsack problem [1].

Let the binary variable $x_{i,t}$ denote if the link l_i is activated at the t -th time-slot or not. Let the binary variable y_t denote if there exists at least one link activated at the t -th time-slot or not. Let g_{ji} denote the path-loss from the sender of link l_j to the receiver of link l_i ; thus g_{ii} denotes the path-loss from the sender of link l_i to its receiver of the same link. Note that P_i denotes the power of link l_i . If the link l_i is active at the t -th time-slot, the SINR requirement can be formulated by the following inequality.

$$P_i g_{ii} x_{i,t} \geq \sigma \left(\sum P_j g_{ji} x_{j,t} + \xi \right)$$

Let $a_i = \frac{P_i \cdot g_{ii}}{\sigma} - \xi$ and $b_{ji} = P_j \cdot g_{ji}$, then this inequality can be written as a knapsack constraint:

$$\sum b_{j,i} x_{j,t} \leq a_i$$

. Thus, for any link l_i and any time-slot t , we have

$$a_i + M_i(1 - x_{i,t}) \geq \sum b_{j,i} x_{j,t}$$

The inequality holds because when $x_{i,t} = 1$, this is exactly the SINR constraint; when $x_{i,t} = 0$, this constraint can be

satisfied for a large value of M_i . To sum up, the integer linear programming is described as follows.

$$\begin{aligned} & \max \sum_{t=1}^n y_t \quad \text{s.t.} \\ & \begin{cases} a_i + M_i(1 - x_{i,t}) \geq \sum b_{j,i} x_{j,t} : \forall i, t \in \{1, 2, \dots, n\}; \\ \sum_{t=1}^n x_{i,t} = 1, \forall i, t \in \{1, 2, \dots, n\}; \\ y_t \geq x_{i,t}, \forall i, t \in \{1, 2, \dots, n\}; \\ x_{i,t} \in \{0, 1\}, \forall i, t \in \{1, 2, \dots, n\}; \\ y_t \in \{0, 1\}, \forall t \in \{1, 2, \dots, n\}. \end{cases} \end{aligned} \quad (2)$$

The second inequalities mean that each link must be activated in one time-slot. The third inequalities reflect the constraint that if a link is activated at some time-slot t , then $y_t = 1$. In this integer linear model, the big number M_i would weaken the continuous relaxation. Even worse, the coefficients of $x_{i,t}$ in the knapsack constraint vary significantly in magnitude and would cause numerical difficulties when solving the problem. Therefore, we reformulate the knapsack constraint by using cover inequality-type cutting planes. A set L of links is called a cover, if $\sum b_{j,i} \geq a_i$. Due to the SINR constraint for link l_i , we can not transmit all links in L simultaneously. Then at most $|L| - 1$ links in L can be active simultaneously and therefore the so-called *cover inequality* holds:

$$\sum_{l_j \in L} x_{j,t} \leq |L| - x_{i,t} : \sum_{l_j \in L} b_{j,i} \geq a_i$$

Note that the inclusion of x_{ij} in the right-hand side restricts the activation of links in L to at most $|L| - 1$, only if link l_i itself is active.

Strengthen Inequalities: We still can not solve the LP in its complete form, as the number of SINR cover inequalities grows exponentially. Let $\{x_{i,t}\}$ denote an integer solution satisfying any subset of SINR cover inequalities. Verifying whether or not $\{x_{i,t}\}$ violates the SINR of any link l_i with $x_{i,t} = 1$ is straightforward. To get stronger inequalities, we generate (7) having a minimum number of elements in the left-hand side, corresponding to so called *minimum cover* for knapsack problems. We minimize the number of interfering nodes we pick before the sum exceeds a_i . Doing so amounts to sorting the elements in b_{ji} in descending order, and following the sorted sequence until the accumulated sum goes above a_i . Denoting the resulting index set by K , then K is a minimal cover. We strengthen it further by a lifting process, *i.e.*, subtracting additional x -variables in the right-hand side based on the conflict constraints. Then the general framework for solving the LP is described as follows:

- 1) Solve the current LP, let $\{x_{i,t}\}$ be the optimal solution.
- 2) If $\{x_{i,t}\}$ is a feasible schedule, then we are done.
- 3) Otherwise, we find possible cover inequalities violated by $\{x_{i,t}\}$ and add to the LP. Go to Step (1).

Input : Set of links $A = \{l_1, l_2, \dots, l_n\}$,
the initial demand function $d \in \mathbb{R}_+^A$.

Output: A fractional link schedule:
 $S = \{(I_j, \gamma_j) : 1 \leq j \leq K\}$.

```

j ← 0 ;
while there exist link(s) in A do
  for k1 = 0, ..., K and k2 = 0, ..., K do
    for i, j ∈ ℤ and the grid gi,j contains links
      from A do
      if i ≡ k1 mod (K + 1) and j ≡ k2
        mod (K + 1) then
        select one link whose sender lies
          within gi,j
    Let all the selected links form a set Ij;
    if Ij ≠ ∅ then
      j ++;
      Compute the minimum demand dmin from
        all links in Ij as follows:
      dmin ← ∞
      for each link e ∈ Ij do
        if d(e) < dmin then
          dmin = d(e);
      Let γj = dmin;
      for each link e ∈ Ij do
        Replace d(e) by d(e) − γj; if d(e) = 0
          then
            Remove the link e from A;
  return A schedule S containing a subset of links in A.

```

Algorithm 2: Fractional Link Scheduling.

VI. DEALING WITH FRACTIONAL DEMAND

In this section, we study the problem FML²S under the physical interference model. We will focus on the linear transmission power control setting. The proposed algorithm can be adapted to some other power control settings as well.

We will construct a fractional link schedule S in an iterative manner. At the beginning of each iteration, we denote the subset of links e in A with $d(e) > 0$ as A' . We use A' as the set of input links and apply a partition-based algorithm to select an independent set I of links (may not be maximal) from A' . Let $\gamma = \min_{e \in I} d(e)$, and add (I, γ) to S . We then update the demand function, by replacing $d(e)$ by $d(e) - \gamma$ for each link e in I .

We repeat this iteration until $d(e) = 0$ (the demand of this link e reaches zero) for every link $e \in A$. The details of the partition-based scheduling algorithm are shown in Algorithm 2.

Note that, after each iteration the number of links with positive demand strictly decreases. Thus, Algorithm 2 will terminate in a finite number of iterations (bounded by $|A|$). The correctness of Algorithm 2 follows from Lemma 1.

Next, we derive the approximation ratio of the proposed scheduling algorithm.

To analyze the performance, we first upper-bound the length of the schedule outputted by Algorithm 2. we then derive a lower bound on the length required for any schedule. By comparing the two parts, we can derive the approximation ratio of the proposed algorithm (Algorithm 2).

For each cell $g_{i,j}$, let $A_{i,j}$ be the set of all links lying inside the cell $g_{i,j}$. Let $D_{i,j}$ be the sum of demands for all links in $A_{i,j}$. Assume the maximum value of $D_{i,j} : \forall i, j \in \mathbb{Z}$ is D (i.e., the maximum total demand in a single cell).

Lemma 4: The proposed algorithm has a length at most $(K+1)^2 \cdot D$.

Proof: Let $k_1 \in \{0, \dots, K\}$ and $k_2 \in \{0, \dots, K\}$. Consider a number pair (k_1, k_2) , for each cell $g_{i,j} : i, j \in \mathbb{Z}$ such that $i \equiv k_1 \pmod{K+1}$ and $j \equiv k_2 \pmod{K+1}$ hold, we select a link arbitrarily from the cell. By Lemma 1, we know that all the selected links can transmit simultaneously. Using this property, we know that after each time-slot, we can satisfy the demand from each cell $g_{i,j}$ by at least one. Thus, to satisfy all demands for all cells $g_{i,j}$ with $i \equiv k_1 \pmod{K+1}$ and $j \equiv k_2 \pmod{K+1}$, we only need a fractional link schedule of length at most D , where D the maximum total demand in a single cell

Now, we consider different $k_1 \in \{0, \dots, K\}$ and $k_2 \in \{0, \dots, K\}$, there are totally at most $(K+1)^2$ different pairs of (k_1, k_2) . For each pair, we only need a fractional link schedule of length at most D . Thus, totally, Algorithm 1 outputs a schedule of length at most $(K+1)^2 \cdot D$. ■

Next, we show the lower-bound of any algorithm for FML²S that is the minimum length of any algorithm.

Lemma 5: Any valid schedule for FML²S has a length at least $\frac{D}{\omega}$.

Proof: By Lemma 3, for any single cell, any valid schedule can satisfy a total demand of at most w in one time-slot. Since there exists a cell which contains a total demand D , any fractional link schedule has a length at least $\frac{D}{\omega}$ to ensure interference-freeness. ■

Theorem 4: The approximation ratio of our algorithm for FML²S is at most $(K+1)^2 \omega$.

Proof: Since our algorithm has a latency at most $(K+1)^2 \cdot D$, at the same time, any valid schedule for FML²S has a latency at least $\frac{D}{\omega}$, thus the approximation ratio of the proposed algorithm can be bounded by $(K+1)^2 \cdot \omega$. So, the theorem holds. ■

VII. LITERATURE REVIEW

Under the physical interference model, the problem of joint scheduling and power control has been well studied. For instance, in [6], [7], optimization models and heuristics for this problem are proposed. In [9], [19], topology control with SINR constraints is studied. In [18], a power-assignment algorithm which schedules a strongly connected set of links in poly-logarithmic time is presented. In [4],

the combined problem of routing and power control is addressed.

In [10], the scheduling problem without power control under physical interference model, where nodes are arbitrarily distributed in Euclidean space, has been shown to be NP-complete. A greedy scheduling algorithm with approximation ratio of $O(n^{1-2/(\Psi(\alpha)+\epsilon)}(\log n)^2)$, where $\Psi(\alpha)$ is a constant that depends on the path-loss exponent α , is proposed in [2]. Notice that this result can only hold when the nodes are distributed uniformly at random in a square of unit area.

In [10], the authors proposed an algorithm with a factor $O(g(L))$ approximation guarantee in arbitrary topologies, where $g(L) = \log \vartheta(L)$ is the diversity of the network. In [5], an algorithm with approximation guarantee of $O(\log \Delta)$ was proposed, where Δ is the ratio between the maximum and the minimum distances between nodes. Obviously, it can be arbitrarily larger than $\vartheta(L)$.

Recently, Goussevskaya *et al.* [11] proposed a method for Maximum Independent Set of Links (MISL). Unfortunately, as observed in Xu and Tang [23], their method [11] only works correctly in absence of the background noise. Wan *et al.* [20] resolved this issue by developing the first correct constant-approximation algorithm. [24] gave a constant-approximation algorithm for the problem of maximum weighted independent set of links under the oblivious power control setting.

Most Recently, Halldorsson *et al.* [13] presented a robustness result, showing that constant parameter and model changes will modify the minimum length link scheduling result only by a constant. [12] and [8] studied the scheduling problem under power control respectively. The minimum latency link scheduling problem has been studied in [22].

VIII. CONCLUSIONS

In this paper, we studied a fundamental problem called Minimum Length Link Scheduling (ML²S) which is crucial to the efficient operations of wireless networks. We focused on the physical interference model and the presence of background noises. We considered this problem under three important transmission power control settings: linear power control, uniform power control and arbitrary power control. We designed a suite of novel scheduling algorithms and conduct explicit complexity analysis to demonstrate the efficiency of the proposed algorithms. In addition, we also investigated the fractional case of the problem ML²S and proposed an efficient greedy algorithm with currently the best approximation ratio.

Some interesting questions are left for future research. The first one is to develop constant approximation scheduling algorithms for uniform transmission power and adjustable transmission power assignment settings. The second one is to extend our algorithms to deal with a more general path loss model.

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APPENDIX

Proof of Theorem 1

We first introduce the *Partition problem*:

Definition 1: Given a set \mathcal{I} of integers, whether we can divide \mathcal{I} into two disjoint subsets \mathcal{I}_1 and \mathcal{I}_2 , such that $\mathcal{I}_1 \cup \mathcal{I}_2 = \mathcal{I}$ and the summations in each subset are equal?

Given an instance of the Partition problem: $\mathcal{I} = \{i_1, \dots, i_n\}$ of integers with $\sum_{j=1}^n i_j = N$; we will construct a ML²S instance with $n + 2$ links $A = \{a_1, \dots, a_{n+2}\}$, where $a_i = \overrightarrow{s_j, r_j}$. Let the parameter $a = \frac{c/\sigma}{N/2}$. For each integer $i_j \in \mathcal{I}$, we set the coordinate of corresponding sender s_j as: $pos(s_i) = \left(\sqrt{\frac{P}{ai_j}}, 0\right)$; and the coordinate of receiver r_i to be $pos(s_i) + (b, 0)$, where $b =$ and $i_{\max} = \max\{i : i \in \mathcal{I}\}$. Finally, we place r_{n+1} and r_{n+2} at the center $(0, 0)$ and s_{n+1}, s_{n+2} at $(0, \pm b)$ respectively. For simplicity, let $\eta = 1, \xi = 0$.

In order to transmit successfully, the SINR constraint at the intended receiver has to be satisfied. In Lemma 6, we will prove that the receivers r_1, \dots, r_n are close enough to their respective senders to guarantee successful transmissions, regardless of the number of other links scheduled simultaneously.

Lemma 6: Let $A_i = \{a_j : 1 \leq j \leq n + 1 \text{ \& } i \neq j\}$. It holds for all $i \leq n$ that the SINR exceeds σ when the link a_i is scheduled concurrently with the set A_i .

Proof: Since the positions of the sender nodes s_1, \dots, s_n depend on the values of i_1, \dots, i_n , we can determine the minimum distance between two sender nodes s_j, s_k : $\|s_j, s_k\| = \|s_j, r_{n+1}\| - \|s_k, r_{n+1}\| = \sqrt{\frac{P}{ai_j}} - \sqrt{\frac{P}{ai_k}} \geq \sqrt{\frac{P}{ai_k}} \cdot \left(\frac{1}{\sqrt{i_{\max}-1}} - \frac{1}{\sqrt{i_{\max}}}\right) = f$.

Thus, any sender (including s_{n+1}) are located at least distance $\|s_j, s_k\| - b$ away. By setting b properly, we can ensure that the successful transmission of a_i . ■

Note that, any schedule to transmit all links needs at least two time-slots, since a_{n+1} and a_{n+2} can never be scheduled simultaneously, we then prove that:

Lemma 7: There exists a solution to the Partition problem if and only if there exists a two-slot schedule for A .

Proof: \Rightarrow : Assume we know two subsets $\mathcal{I}_1, \mathcal{I}_2 \subset \mathcal{I}$, whose elements sum up to $N/2$. To construct a two-slot schedule, $\forall i_j \in \mathcal{I}_1$, we assign the link a_j to the first time slot, along with a_{n+1} , and assign the remaining links to the second time slot. We then check the correctness of our schedule. The signal power that r_{n+1} receives from s_{n+1} is c . The interference r_{n+1} experiences from each sender s_j is ai_j , which results in the SINR at r_{n+1} of at least σ . This fact, in combination with Lemma 6, proves that our schedule guarantees interference-freeness for all links.

\Leftarrow : if no solution to the Partition problem exists, this implies that for every partition of \mathcal{I} into two subsets, the sum of one set is greater than $N/2$. Assume we could still find a schedule with only two slots. Since the receivers r_{n+1} and r_{n+2} are at the same position, they have to be assigned to different slots. Assume r_{n+1} is assigned to the set with sum greater than $N/2$, then the SINR at r_{n+1} is below σ , which prevents the correct reception of the signal from s_{n+1} , this causes contradiction. ■

Proof of Lemma 1

Proof: Consider any link $a = (u, v)$. The wanted signal strength is

$$c \|a\|^\beta \cdot \eta \|a\|^{-\kappa} = c\eta \|a\|^{\beta-\kappa} \geq c\eta R^{\beta-\kappa}.$$

Consider any link $a' = (u', v')$ in I other than a . We have $\|u'u\| \geq K\ell$. Therefore,

$$\|u'v\| \geq \|u'u\| - \|uv\| \geq K\ell - R = \left(K/\sqrt{2} - 1\right) R.$$

The total interference to a from all other links in I is at most

$$\begin{aligned} & \sum_{(x,y) \in \mathbb{Z}^2 \setminus \{(0,0)\}} cR^\beta \cdot \eta \left(\sqrt{x^2 + y^2} \cdot \left(K/\sqrt{2} - 1\right) R\right)^{-\kappa} \\ &= c\eta R^{\beta-\kappa} \left(K/\sqrt{2} - 1\right)^{-\kappa} \sum_{(x,y) \in \mathbb{Z}^2 \setminus \{(0,0)\}} \left(\sqrt{x^2 + y^2}\right)^{-\kappa} \\ &\leq 4c\eta R^{\beta-\kappa} \left(K/\sqrt{2} - 1\right)^{-\kappa} \left(\sum_{i=1}^{\infty} i^{-\kappa} + \sum_{x=1}^{\infty} \sum_{y=1}^{\infty} \left(\sqrt{x^2 + y^2}\right)^{-\kappa}\right) \\ &\leq 4c\eta R^{\beta-\kappa} \left(K/\sqrt{2} - 1\right)^{-\kappa} \left(\frac{\kappa(1 + 2^{-\frac{\kappa}{2}})}{\kappa - 1} + \frac{\pi 2^{-\kappa/2}}{2(\kappa - 2)}\right) \\ &= 4\tau c\eta R^{\beta-\kappa} \left(K/\sqrt{2} - 1\right)^{-\kappa}, \end{aligned}$$

where

$$\tau = \frac{\kappa(1 + 2^{-\frac{\kappa}{2}})}{\kappa - 1} + \frac{\pi 2^{-\kappa/2}}{2(\kappa - 2)}.$$

Thus the SINR at the receiver of the link is at least

$$\frac{c\eta R^{\beta-\kappa}}{\xi + 4\tau c\eta R^{\beta-\kappa} \left(K/\sqrt{2} - 1\right)^{-\kappa}} \geq \sigma$$

since

$$K/\sqrt{2} - 1 \geq \left((4\tau)^{-1} \left(\sigma^{-1} - \xi(c\eta)^{-1} R^{\kappa-\beta}\right)\right)^{-1/\kappa}$$

■